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UNIQUENESS OF THE SOLUTION FOR A PARABOLIC-HYPERBOLIC EQUATION WITH FRACTIONAL ORDER CAPUTO OPERATOR IN TWO-DIMENSIONAL DOMAIN ON A BOUNDARY-VALUE PROBLEM

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Abstract. In this paper we study a new problem for a parabolic-hyperbolic equation with fractional order Caputo operator in two-dimensional domain. There are many works devoted to study problems for the second order mixed parabolic-hyperbolic and elliptic-hyperbolic type equations in rectangular domains with two gluing conditions with respect to second argument and with boundary value conditions on all borders of the domain. In studying the unique solvability of this problem, it becomes necessary to specify an additional condition on the hyperbolic boundary of the domain. For this reason, the considering problem became unresolved in an arbitrary rectangular domain. In this paper, we were able to remove this restriction by setting three gluing conditions for the second argument.

2. PROBLEM FORMULATION

We consider the following mixed equation

$$\begin{cases} cD_t^{\alpha}u - \Delta u = f(x,y,t)\ , t > 0 \\ u_{tt} - \Delta u = f(x,y,t)\ , t < 0 \end{cases} \quad (1)$$

in rectangular domain $D = \{(x,y): 0 < x < 1, 0 < y < 1\ , -p < t < q\}$, where $p > 0, q > 0$ are given real constants and $f(x,y)$ is given continuous function,

$$cD_t^{\alpha} = I_{0+}^{1-\alpha}u_t = \frac{1}{\Gamma(1-\alpha)}\int_0^t \frac{u_\tau(x,y,\tau)d\tau}{(t-\tau)^{\alpha}} \quad (2)$$

is the Caputo differential operator of fractional order $0 < \alpha < 1$ [7]. Notice, that Caputo differential operator expresses by the aid of fractional order Riemann–Liouville integral [7]. where $\Gamma(\alpha)$is Euler gamma function [27],

We introduce following denotations: $J = \{(x,y): 0 < x < 1, 0 < y < 1, t = 0\}$, $D_1 = D \cap \{x > 0, y > 0, t > 0\}$, $D_2 = D \cap \{x > 0, y > 0, t < 0\}$, $D = D_1 \cup D_2 \cup J$.

In the domain D we study the following problem.

And let $\Omega = (0,1) \times (0,1)$

Problem IU. To find a function $u(x,y)$ with following properties:

1)$u(x,y,t) \in \bar{C}(D) \cap C^1(D_2 \cup J), t^{1-\alpha}u_t(x,y,t), t^{2-\alpha}u_{tt}(x,y,t) \in C(D_1 \cup J)$;

2) $u_{tt} \in C(D_2 \cup J), u_{xx} \in C(D_1 \cup D_2 \cup J), u_{yy}\, C(D_1 \cup D_2 \cup J)\ cD_{0y}^{\alpha}u \in C(D_1 \cup J)$ and satisfies to equation (1) in the domains $D_j(j = 1, 2)$;

3) on the plane J take place the following gluing conditions

$$\lim_{t\to+0} u = \lim_{t\to-0} u\ , \quad (3)$$

$$\lim_{t\to+0} t^{(1-\alpha)}u_t = \lim_{t\to-0} u_t\ , \quad (4)$$

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$$\lim_{t\to+0} t^{2-\alpha}u_{tt} = \lim_{t\to-0} u_{tt} \ , \tag{5}$$

4) $u(x,y,t)$ satisfies the following boundary conditions:

$$u|_{x=0} = u|_{x=1} = 0, u|_{y=0} = u|_{y=1} = 0 \tag{6}$$

3. UNIQUENESS OF SOLUTION

Theorem. The solution of the above formulated problem IU is unique, if it exists. Let us suppose that $f\ (x,y)\ =\ 0$ in D. Then we prove, that corresponding homogeneous problem has only trivial solution. It is well known, that the functions

$$\begin{gathered} X_n\ (x)\ =\ \sqrt{2}\ sin\ \mu_n x \\ Y_m\ (x) =\ \sqrt{2} sin\ \nu_m y, \\ \lambda_{mn}^2 = \mu_n^2 + \upsilon_m^2 \\ \mu_n = \pi n;\ \ \upsilon_m = \pi m \end{gathered} \tag{11}$$

form complete orthonormal system in the space $L^2(\Omega)$. We consider the integrals

$$\theta_{mn}(t) = \int_0^1 \int_0^1 u(x,y,t)\ X_n(x) Y_m(y) dx\ dy\ \ , t > 0 \tag{12}$$

$$\beta_{mn}(t) = \int_0^1 \int_0^1 u(x,y,t)\ X_n(x) Y_m(y) dx\ dy\ , t < 0 \tag{13}$$

By virtue of (12) and (13), from homogeneous equation (1) we have

$$D_{0t}^{\alpha}\theta_{mn}(t) = 2\int_0^1 \int_0^1 [D_{0t}^{\alpha} u(x,y,t)]\ X_n(x) Y_m(y) dx\ dy$$

$$= 2\int_0^1 \int_0^1 \Delta u(x,y,t)\ X_n(x) Y_m(y) dx\ dy\ \ , 0 < t < q$$

$$\beta_{mn}''(t) = 2\int_0^1 \int_0^1 [u_{tt}(x,y,t)] X_n(x) Y_m(y) dx\ dy$$

$$= 2\int_0^1 \int_0^1 \Delta u(x,y,t) X_n(x) Y_m(y) dx\ dy, -p < t < 0$$

From the last, integrating two times by parts the expressions (5)–(8), we obtain

$$D_{0t}^{\alpha}\theta_{mn}(t) + \lambda_{mn}^2 \theta_{mn}(t) = 0, 0 < t < q\ \ , \tag{14}$$

$$\beta_{mn}(t) +\ \lambda_{mn}^2 \beta_{mn}(t) =\ 0, -p\ <\ t\ <\ 0, \tag{15}$$

$$\theta_{mn}(+0) =\ \beta_{mn}(-0), \tag{16}$$

$$\lim_{t\to+0} t^{1-\alpha}\theta'_{mn}(t) = \lim_{t\to-0} \beta'_{mn}(t), \tag{17}$$

$$\lim_{t\to+0} t^{2-\alpha}\theta''_{mn}(t) = \lim_{t\to-0} \beta''_{mn}(t)\ \ , \tag{18}$$

Differential equations (14) and (15) have general solutions in the forms [7, page 17]

$$\theta_{mn}(t) = \omega_{mn} E_{\frac{1}{\alpha}}(-\lambda_{mn}^2 t^{\alpha}, 1), 0 < t < q, \tag{19}$$

$$\beta_{mn}(t)\ =\ \delta n\ cos\ \lambda_{mn} t\ +\ \gamma_{mn} sin\ \lambda_{mn} t, -p\ <\ t\ <\ 0, \tag{20}$$

respectively, where $\delta_{mn}, \gamma_{mn}, \omega_{mn}$ are any constants, and $E_{\frac{1}{\alpha}}(z,\mu)$ is well known Mittag-Leffler function, which represented as [7]:

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$$E_{\alpha,\mu}(z) \equiv E_{\frac{1}{\alpha}}(z,\mu) = \sum_{i=0}^{\infty} \frac{z^i}{\Gamma(\alpha i+\mu)}\ ,\mu > 0 \qquad (21)$$

Substituting (19) and (20) into (16)–(18) with properties of Mittag-Leffler function [2], [7, page 13,
form. (1.1.12)]

$$E_{\frac{1}{\alpha}}(z) = 1 + zE_{\frac{1}{\alpha}}(z,\alpha+1), \qquad (22)$$

we have $\delta_{\mathrm{mn}} = \omega_{\mathrm{mn}}\ ,\gamma_{mn} = -\frac{\lambda_{mn}}{\Gamma(\alpha)}\omega_{mn},\ \ \lambda_{mn}^2\left[\frac{(1-\alpha)}{\Gamma(\alpha)}+1\right]\omega_{mn} = 0.$

Hence, we obtain $\omega_{mn} = \delta_{mn} = \gamma_{mn} = 0$. Consequently, $\theta_{mn}(t) = 0, \beta_{mn}(t) = 0$. Then, from the equalities (12) and (13) imply, that

$$\theta_{mn}(t) = \int_0^1 \int_0^1 u(x,y,t)\ X_n(x)Y_m(y)dx\,dy = 0, t > 0$$

$$\beta_{mn}(t) = \int_0^1 \int_0^1 u(x,y,t)\ X_n(x)Y_m(y)dx\,dy = 0, t < 0$$

Further, by virtue of completeness of $X_n(x), Y_m(y)$ we deduce that $u(x,y,t) \equiv 0$ in *D*, i.e. a solution of the investigated problem is unique. The Theorem is proved

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O'QUV ROBOTOTEXNIKASIDA QO'LLANILADIGAN KONSTRUKTOR TIZIMINI LOYIHALASHNING AHAMIYATI

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Ta'lim robototexnikasi so'nggi yillarda STEM (fan, texnologiya, muhandislik va matematika) ta'limini yaxshilashning istiqbolli vositasi sifatida katta e'tibor qozondi. Ta'lim robototexnikasining asosiy komponenti konstruktor tizimi bo'lib, o'quvchilarga robot tuzilmalarini yaratish va dasturlash imkonini beradigan jismoniy yoki virtual qurilish bloklari to'plamidir. Ushbu maqola ta'lim robototexnikasida samarali konstruktor tizimini loyihalashning ahamiyatini o'rganadi. U konstruktor tizimlari bilan bog'liq afzalliklar va muammolarni ta'kidlab, ularning o'quvchilarning ta'lim natijalari, faolligi va ijodkorligiga ta'siri haqida tushuncha beradi. Bundan tashqari, u o'quv robototexnikasida tajriba o'tkazish, muammolarni hal qilish ko'nikmalari va tanqidiy fikrlashni rivojlantirishga yordam beradigan konstruktor tizimini loyihalash uchun asosiy fikrlarni taqdim etadi.

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