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REZYUME

Mazkur maqolada to'rt o'lchamli Li bo'lmagan filiform Leybnits algebralarining ichki lokal differensiallashlari o'rganilgan. $F^1(\alpha_4,\theta)$ algebrasining har bir lokal ichki differensiallashi ichki differensiallash ekanligi isbotlangan.

РЕЗЮМЕ

В статье изучены локальные внутренние дифференцирования четырехмерных филиформных не лиевых алгебр Лейбница. Доказано что, всякое локальное внутреннее дифференцирование алгебры $F^1(\alpha_4,\theta)$ кроме случая $\alpha_4=\theta\neq 0$ является внутренним дифференцированием и в случае $\alpha_4=\theta\neq 0$ на алгебре $F^1(\alpha_4,\theta)$ существует локальное внутреннее дифференцирование которое не является внутренним дифференцированием.

SUMMARY

In this article, local inner derivations are studied of four-dimensional filiform of non-Lie Leibniz algebras. It is proved that every local inner derivation of the algebra $F^1(\alpha_4,\theta)$ is an inner derivation.

LOCAL INNER DERIVATIONS ON THREE-DIMENSIONAL LIE ALGEBRAS

A.K.Alauadinov – *doctor of Philosophy in Physical and Mathematical Sciences*
Karakalpak state university name after Berdax
Ch.I.Jumaeva – *master*
V.I.Romanovskiy institute of Mathematics Uzbekistan Academy of Sciences

Tayanch so'zlar: Li algebrasi, differensiallash, ichki differensiallash, lokal ichki differensiallash.
Ключевые слова: алгебра Ли, дифференциация, внутренняя дифференциация, локальная внутренняя дифференциация.
Key words: Lie algebra, derivation, inner derivation, local derivation, local inner derivation.

1. Introductions. Currently, various types of derivations over algebras are being studied, for example, derivations, local derivations, 2-local derivations, almost inner derivations, etc. Local derivations were first considered in the work of R. Kaidison in 1990 [10] and, independently in the work of D. Larson and A. Surur [11]. In these papers, some conditions are indicated under which local derivations is derivations. R.Kaidison's paper considered local derivations on von Neumann algebras and in some polynomial algebras. L.Molnár [12] introduced the definition of local inner derivations on standard operator algebras.

The first results concern to local and 2-local derivations and automorphisms on finite-dimensional Lie algebras over algebraically closed field of zero characteristic were obtained in [2, 3, 5] and [7]. Namely, in [5] it is proved that every 2-local derivation on a semi-simple Lie algebra L is a derivation and that each finite-dimensional nilpotent Lie algebra with dimension larger than two admits 2-local derivation which is not a derivation. In [2] the authors have proved that every local derivation on a semi-simple Lie algebras is a derivation and gave examples of nilpotent finite-dimensional Lie algebras with local derivations which are not derivations. Concerning 2-local automorphism, Z.Chen and D.Wang in [7] prove that if L is a simple Lie algebra of type A_l, D_l or E_k, (k = 6,7,8) over an algebraically closed field of characteristic zero, then every 2-local automorphism of *L* is an automorphism. Finally, in [3] Sh.A.Ayupov and K.K.Kudaybergenov generalized this result of [7] and proved that every 2-local automorphism of a finite-dimensional semi-simple Lie algebra over an algebraically closed field of characteristic zero is an automorphism. Moreover, they show also that every nilpotent Lie algebra with finite-dimension larger than two admits 2-local automorphisms which is not an automorphism. Local automorphisms of certain finite-dimensional simple Lie and Leibniz algebras are investigated in [4]. Almost inner derivations of Lie algebras were introduced by C.S. Gordon and E.N. Wilson [8] in the study of isospectral deformations of compact manifolds. Almost inner derivations of nilpotent, some solvable Lie algebras and some nilpotent Leibniz algebras were studied in the papers by [6] and [1].

2. Preliminaries. To begin with, recall the definition of Lie algebras.

Definition 2.1. An algebra L over field $\mathbb{F}$ is called a *Lie algebra* if its multiplication satisfies the identities:

1) $[x,x]=0$,
2) $[x,[y,z]]+[y,[z,x]]+[z,[x,y]]=0$,

for all x,y,z in L.

The product $[x,y]$ is called the bracket of x and y. Identity 2) is called the Jacobi identity.

Let L be a finite-dimensional Lie algebra. For Lie algebra L we consider the following central and derived series:

$$L^1=L,\ L^i=\left[L^{i-1},L\right],\ i\geq 1,$$

$$L^{[1]}=L,\ L^{[k]}=\left[L^{[k-1]},L^{[k-1]}\right],\ k\geq 1.$$

A Lie algebra L is *nilpotent (solvable)* if there exists $m\geq 1$ such that $L^m=0$ ($L^{[m]}=0$).

Definition 2.2. A *derivation* on a Lie algebra L is a linear map $D:L\to L$ which satisfies the Leibniz rule:

$$D([x,y])=[D(x),y]+[x,D(y)] \qquad (2.1)$$

for any $x, y \in L$. The set of all derivations of a Lie algebra denoted by $Der(L)$. Let a be an element of a Lie algebra L. Consider the operator of $ad_a : L \to L$ defined by $ad_a(x) = [x, a]$. This operator is a derivation and called *inner derivation.* The set of all inner derivations of a Lie algebra denoted by $InDer(L)$.

Definition 2.3. A linear operator Δ is called a *local derivation* if for any $x \in L$, there exists a derivation $D_x : L \to L$ (depending on x) such that $\Delta(x) = D_x(x)$.

Definition 2.4. A linear operator Δ is called a *local inner derivation* if for any $x \in L$, there exists a inner derivation $ad_x : L \to L$ (depending on x) such that $\Delta(x) = ad_x(x)$.

We present the following theorem which gives a classification of arbitrary three-dimensional Lie algebras.

Theorem 2.1. [9]. An arbitrary three-dimensional Lie algebra is isomorphic to one of the following algebras:

L_0 : abelian;

$L_1 : [e_1, e_2] = e_3;$

$L_2 : [e_1, e_2] = e_1;$

$L_3 : [e_1, e_2] = e_2, [e_1, e_3] = e_2 + e_3;$

$L_4 : [e_1, e_2] = e_2, [e_1, e_3] = \lambda e_3, \lambda \in C^*, |\lambda| \le 1;$

$L_5 : [e_1, e_2] = e_3, [e_1, e_3] = -2e_1, [e_2, e_3] = 2e_2.$

3. Main results. In this section we will consider local inner derivations of three-dimensional Lie algebras.

The following theorem is the main result of this work.

Theorem 3.1. *Any local inner derivation on three-dimensional Lie algebras is an inner derivation.*

Proof. We will check whether the local inner derivations of these six algebras are inner derivations.

The algebra L_0. First, consider the algebra L_0 from Theorem 2.1. Inner derivations on the algebra L_0 are zero. Therefore, any local inner derivation is also zero.

For the algebras L_1 and L_2, we check the equality $\Delta(e_i) = ad_{e_i}(e_i)$ $(i = 1,2,3)$ for the basis $\{e_1, e_2, e_3\}$.

The algebra L_1. For $x = \sum_{i=1}^{3} x_i e_i \in L_1$ exists element $a = \sum_{i=1}^{3} a_i e_i \in L_1$ such that

$$ad_a(x) = [x, a] = [x_1e_1 + x_2e_2 + x_3e_3, a_1e_1 + a_2e_2 + a_3e_3] = $$
$$= (x_1a_2 - x_2a_1)e_3;$$

Hence, matrix of the inner derivations of L_1 has the form

$$ad = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ a_2 & -a_1 & 0 \end{pmatrix}.$$

Let

$$\Delta = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \quad (3.1)$$

the local inner derivation of algebra L_1.

By definition of local inner derivation, checking the equality $\Delta(e_i) = ad_{e_i}(e_i)$ $(i = 1,2,3)$ for the basis $\{e_1, e_2, e_3\}$, we obtain the following:

- For e_1: $\Delta(e_1) = a_{11}e_1 + a_{21}e_2 + a_{31}e_3$, $ad_{e_1}(e_1) = a_2^{(e_1)}e_3$.

Comparing the coefficients of the basic elements, we have

$$a_{11} = a_{21} = 0, \ a_{31} = a_2^{(e_1)};$$

- For e_2: $\Delta(e_2) = a_{12}e_1 + a_{22}e_2 + a_{32}e_3$, $ad_{e_2}(e_2) = -a_1^{(e_2)}e_3$. Then we get

$$a_{12} = 0, a_{22} = 0, a_{32} = -a_1^{(e_2)};$$

- For e_3: $\Delta(e_3) = a_{13}e_1 + a_{23}e_2 + a_{33}e_3$; $ad_{e_3}(e_3) = 0$. Then we obtain

$$a_{13} = a_{23} = a_{33} = 0.$$

Then (3.1) has the following form

$$\Delta = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ a_{31} & a_{32} & 0 \end{pmatrix}.$$

From the last obtained we have $\Delta(e_1) = a_2^{(e_1)}e_3$, $\Delta(e_2) = -a_1^{(e_2)}e_3$. Since, operator Δ is linear, then

$$\Delta(x) = \Delta(x_1e_1 + x_2e_2 + x_3e_3) = (a_2^{(e_1)}x_1 - a_1^{(e_2)}x_2)e_3.$$

This means that the operator Δ is an inner derivation.

The algebra L_2. Repeating the previous technique for this algebra, we get that inner and local inner derivations have the following forms respectively

$$ad = \begin{pmatrix} a_2 & -a_1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \Delta = \begin{pmatrix} a_{11} & a_{12} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

This means that the operator Δ is an inner derivation.

Note that in this algebra, every local inner derivation is an inner derivation.

For the algebras L_3 and L_4, we check the equality of $\Delta(x) = ad_x(x)$ for values of x equal to e_1, e_2, e_3 and $e_2 + e_3$

The algebra L_3. Let $x = \sum_{i=1}^{3} x_i e_i \in L_3$. Then for element $x \in L_3$ exists element $a = \sum_{i=1}^{3} a_i e_i \in L_3$ such that

$$ad_a(x) = [x, a] = [x_1e_1 + x_2e_2 + x_3e_3, a_1e_1 + a_2e_2 + a_3e_3] = $$
$$= (x_1a_2 + x_1a_3 - x_2a_1 - x_3a_1)e_2 + (x_1a_3 - x_3a_1)e_3.$$

Hence, matrix of the inner derivations of L_3 has the next form

$$ad = \begin{pmatrix} 0 & 0 & 0 \\ a_2 + a_3 & -a_1 & -a_1 \\ a_3 & 0 & -a_1 \end{pmatrix}$$

Let

$$\Delta=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}.$$

the local inner derivation of the algebra L_3.

Checking the equality of $\Delta(x)=ad_x(x)$ for values of x equal to e_1,e_2,e_3 and e_2+e_3, we obtain the following:

- For e_1: $\Delta(e_1)=a_{11}e_1+a_{21}e_2+a_{31}e_3$, $ad_{e_1}(e_1)=\left(a_2^{(e_1)}+a_3^{(e_1)}\right)e_2+a_3^{(e_1)}e_3$, and so

$$a_{11}=0, a_{21}=a_2^{(e_1)}+a_3^{(e_1)}, a_{31}=a_3^{(e_1)}.$$

- For e_2: $\Delta(e_2)=a_{12}e_1+a_{22}e_2+a_{32}e_3$, $ad_{e_2}(e_2)=-a_1^{(e_2)}e_2$. Then we get

$$a_{12}=0, a_{22}=-a_1^{(e_2)}, a_{32}=0.$$

- For e_3: $\Delta(e_3)=a_{13}e_1+a_{23}e_2+a_{33}e_3$, $ad_{e_3}(e_3)=-a_1^{(e_3)}e_2-a_1^{(e_3)}e_3$. Then we obtain

$$a_{13}=0, a_{23}=a_{33}=-a_1^{(e_3)}.$$

- For e_2+e_3: $\Delta(e_2+e_3)=(a_{22}+a_{23})e_2+a_{23}e_3$, $ad_{e_2+e_3}(e_2+e_3)=-2a_1^{(e_2+e_3)}e_2-a_1^{(e_2+e_3)}e_3$ and

$$a_{22}=a_{23}.$$

Then local inner derivation of the algebra L_3 has the following form

$$\Delta=\begin{pmatrix} 0 & 0 & 0 \\ a_{21} & a_{22} & a_{22} \\ a_{31} & 0 & a_{22} \end{pmatrix}$$

This means that the operator Δ is an inner derivation.

The algebra L_4. As in the case of algebra L_3, it is shown that

$$ad=\begin{pmatrix} 0 & 0 & 0 \\ a_2 & -a_1 & 0 \\ \lambda a_3 & 0 & -\lambda a_1 \end{pmatrix}, \quad \Delta=\begin{pmatrix} 0 & 0 & 0 \\ a_{21} & a_{22} & 0 \\ \lambda a_3^{(e_1)} & 0 & \lambda a_{22} \end{pmatrix}.$$

This means that the operator Δ is an inner derivation.

The algebra L_5. Let $x=\sum_{i=1}^{3}x_ie_i\in L_5$. Then for element $x\in L_5$ exists element $a=\sum_{i=1}^{3}a_ie_i\in L_5$ such that

$$ad_a(x)=[x,a]=[x_1e_1+x_2e_2+x_3e_3, a_1e_1+a_2e_2+a_3e_3]=$$
$$=(2x_3a_1-2x_1a_3)e_1+$$
$$+(2x_2a_3-2x_3a_2)e_2+(x_1a_2-x_2a_1)e_3.$$

Hence, matrix of the inner derivations of L_5 has the form

$$ad=\begin{pmatrix} -2a_3 & 0 & 2a_1 \\ 0 & 2a_3 & -2a_2 \\ a_2 & -a_1 & 0 \end{pmatrix}.$$

Let

$$\Delta=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}.$$

the local inner derivation of algebra L_5.

Checking the equality of $\Delta(x)=ad_x(x)$ for values of x equal to $e_1,e_2,e_3,e_1+e_2,e_1+e_3$ and e_2+e_3, we obtain the following:

- For e_1: $\Delta(e_1)=a_{11}e_1+a_{21}e_2+a_{31}e_3$, $ad_{e_1}(e_1)=-2a_3^{(e_1)}e_1+a_2^{(e_1)}e_3$. Comparing the coefficients of the element e_2, we have

$$a_{21}=0.$$

- For e_2: $\Delta(e_2)=a_{12}e_1+a_{22}e_2+a_{32}e_3$, $ad_{e_2}(e_2)=2a_3^{(e_2)}e_2-a_1^{(e_2)}e_3$. Comparing the coefficients of the element e_1, we have

$$a_{12}=0.$$

- For e_3: $\Delta(e_3)=a_{13}e_1+a_{23}e_2+a_{33}e_3$, $ad_{e_3}(e_3)=2a_1^{(e_3)}e_1-2a_2^{(e_3)}e_2$. Comparing the coefficients of the element e_3, we have

$$a_{33}=0.$$

- For e_1+e_2: $\Delta(e_1+e_2)=a_{11}e_1+a_{22}e_2+(a_{31}+a_{32})e_3$, $ad_{e_1+e_2}(e_1+e_2)=-2a_3^{(e_1+e_2)}e_1+$ $+2a_3^{(e_1+e_2)}e_2+\left(a_2^{(e_1+e_2)}-a_1^{(e_1+e_2)}\right)e_3$. Comparing the coefficients of the elements e_1 and e_2, we have

$$a_{11}=-2a_3,\ a_{22}=2a_3 \text{ i.e. } a_{22}=-a_{11}.$$

- For e_1+e_3: $\Delta(e_1+e_3)=(a_{11}+a_{13})e_1+a_{23}e_2+a_{31}e_3$, $ad_{e_1+e_3}(e_1+e_3)=\left(2a_1^{(e_1+e_3)}-2a_3^{(e_1+e_3)}\right)e_1-2a_2^{(e_1+e_3)}e_2+a_2^{(e_1+e_3)}e_3$.

Comparing the coefficients of the elements e_2 and e_3, we have

$$a_{23}=-2a_2, a_{31}=a_2 \text{ i.e. } a_{23}=-2a_{31}.$$

- For e_2+e_3: $\Delta(e_2+e_3)=a_{13}e_1+(-a_{11}+a_{23})e_2+a_{32}e_3$, $ad_{e_2+e_3}(e_2+e_3)=2a_1^{(e_2+e_3)}e_1+$ $+\left(2a_3^{(e_2+e_3)}-2a_2^{(e_2+e_3)}\right)e_2-a_1^{(e_2+e_3)}e_3$. Comparing the coefficients of the elements e_1 and e_3, we have

$$a_{13}=2a_1, a_{32}=-a_1 \text{ i.e. } a_{13}=-2a_{32}.$$

Then local inner derivation of algebra L_5 has the following form

$$\Delta=\begin{pmatrix} a_{11} & 0 & -2a_{32} \\ 0 & -a_{11} & -2a_{31} \\ a_{31} & a_{32} & 0 \end{pmatrix}.$$

This means that the operator Δ is an inner derivation.

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REZYUME

Ushbu maqolada biz uch oʻlchovli Li algebralarining mahalliy ichki hosilalarini oʻrganamiz. Biz uch oʻlchovli Li algebralarining har qanday mahalliy ichki hosilasi ichki hosila ekanligini koʻrsatamiz.

РЕЗЮМЕ

В этой статье мы исследуем локальные внутренние дифференцирования трехмерных алгебр Ли. Мы показываем, что любое локальное внутреннее дифференцирование трехмерных алгебр Ли является внутренним дифференцированием.

SUMMARY

In this paper we investigate local inner derivations of three-dimensional Lie algebras. We show that any local inner derivation of three-dimensional Lie algebras is an inner derivation.

УДК 517.91/943

О ПОСТРОЕНИИ АСИМПТОТИЧЕСКИХ РЕШЕНИЙ СИСТЕМЫ НЕЛИНЕЙНЫХ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ С МАЛЫМ ПАРАМЕТРОМ ПРИ ПРОИЗВОДНЫХ С ПЕРЕМЕННЫМ ОТКЛОНЕНИЕМ АРГУМЕНТА

А.Г.Алишев – *кандидат физико-математических наук*
Джизакский государственный педагогический институт
Ш.А.Алишев – *преподаватель*
Джизакский политехнический институт

Таянч сўзлар: вектор, Тейлор, тенглама, коэффициент, илдиз, асимптотика, дифференциал, тизим, расмий, парчаланиш, кетма-кетлик, секин ўзгарувчан.
Ключевые слова: вектор, Тейлор, уравнения, коэффициент, корень, асимптотика, дифференциал, система, формальный, разложения, ряд, медленно меняющийся.
Key words: vector, Taylor, equation, coefficient, root, asymptotics, differential, system, formal, expansion, series, slowly changing.

Рассмотрим систему

$$\varepsilon\frac{d^2x}{dt^2}=A(\tau,\varepsilon)x+\varepsilon B(\tau,\varepsilon)\frac{d^2x(t-\Delta(\tau),\varepsilon)}{dt^2}+\varepsilon f(\tau,x,\varepsilon) \quad (1)$$

где $x(t,\varepsilon)$, $f-n-$мерные векторы, $A(\tau,\varepsilon)$, $B(\tau,\varepsilon)-$ действительные $(n\times n)$ матрицы. $\Delta(\tau)\geq 0$, переменные отклонения, $0\leq\tau=\varepsilon t\leq L<+\infty-$медленное время, $\varepsilon>0$ малые параметры.

В [1] рассмотрены асимптотические разложения решений системы линейных дифференциальных уравнений первого порядка нейтрального типа, когда характеристическое уравнение имеет отличная от нуля корня.

В настоящей работе рассматривается вопрос построения решения системы (1) при наличии нулевого корня уравнения (3), т.е. так называемый критический случай [2,3]. Этот случай для системы вида (1) в литературе не рассматривалась.

Теорема 1. Пусть выполняется условия:

а) матрицы $A_s(\tau)$, $B_s(\tau)$ $(s=0,1,...)$ и функция $\Delta(\tau)>0$ при $\tau\in[0,L]$, а вектор $f(\tau,x,\varepsilon)$ в области $P(\tau,x,\varepsilon)=P(\tau,x)\times(0<\varepsilon\leq\varepsilon_0]$, где $P(\tau,x)-$ некоторая область пространство переменных (τ,x), неограниченно дифференцируемы;

б) при $\tau\in[0,L]$, $w_1(\tau)=0$, $\operatorname{Re} w_q(\tau)\leq 0$ $q=\overline{2,n}$;

в) $(\psi,A_1(\tau)\varphi)\neq 0,$ $(\psi,A_1(\tau)\varphi)\neq(\psi,f_x(\tau)\varphi),$ $\forall\tau\in[0,L]$ (4)

где $\varphi\in N(A_0(\tau)),$ $\psi\in N(A_0^*(\tau))-A_0^*(\tau)-$ комплексно сопряжённая матрица к матрице $A_0(\tau)$ (см[3,4]). Тогда уравнение (1) имеет формальное частное решение вида

$$x(t,\varepsilon)=\sum_{S=0}^{\infty}\mu^S u_S(\tau),\mu=\sqrt{\varepsilon}. \quad (5)$$

Теорема 2. Пусть выполняется условие а теоремы 1, а также матрица $A_0(\tau)$ имеет полупростое нулевое собственное значение кратности $p<n$ (при $p=n$ матрица $A_0(\tau)\equiv 0$). Тогда система (1) имеет решение вида (5).

Теорема 3. Если выполняется условие а) теоремы 1, и характеристическое уравнение (3) системы (1) имеет один нулевой корень кратности n,с корневым подпро-