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MATHEMATICAL ANALYSIS AND ITS APPLICATIONS IN MODERN MATHEMATICAL PHYSICS

PART I

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MATHEMATICAL ANALYSIS AND ITS APPLICATIONS IN MODERN MATHEMATICAL PHYSICS

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The collection of abstracts of the reports of the international scientific conference "Mathematical analysis and its applications in modern mathematical physics" contains scientific reports on the following areas: Mathematical analysis, differential equations, mathematical physics, algebra, geometry, numerical mathematics, mathematical modeling, probability theory, mathematical statistics, mathematical engineering and information technologies .

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CONTENTS

SECTION I. MATHEMATICAL ANALYSIS

On the continuation of solution of the generalized Cauchy-Riemann system with quaternion parameter

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In this paper, we present an explicit formula for the continuation of the solution of the Cauchy problem for a generalized Cauchy-Riemann system with quaternion parameter[1]

$$\alpha_0 f_0 - divf - < f, \overrightarrow{\alpha} >= 0, \;\; gradf_0 + rotf + [f \times \overrightarrow{\alpha}] + f_0 \overrightarrow{\alpha} + \alpha_0 f = 0, \tag{1}$$

. Ω is a bounded simply connected domain in R^3 with boundary $\partial\Omega$ composed of a compact connected part T of the plane $y_3 = 0$ and a smooth Lyapunov surface S lying in the half-space $y_3 > 0$, with $\bar{\Omega} = \Omega \cup \partial D$, $\partial\Omega = S \cup T$. As to S, we assume that each ray issuing from any point x of the domain Ω intersects this surface at most l points. The solution of the Cauchy problem will be constructed in the domain Ω for the case in which the Cauchy data are given on a part S of the boundary. The Cauchy problem for a generalized Cauchy-Riemann system with quaternion parameter is an ill-posed problem (see Hadamard's example in [2. p.39 (Russain transl.)]).

We assume that the solution of the problem exists (then it is unique [3, p.58]) and the exact Cauchy data are given. Under these conditions, we establish an explicit continuation formula which is an analog of the classical Riemann-Volterra-Hadamard formula for solving the Cauchy problem in the theory of hyperbolic equations. An explicit regularization formula is proposed for the conditions given above hold and, instead of the Cauchy data, their continuous approximations with prescribed deviation in a uniform metric are given under the condition that the solution is bounded by a positive number on a part T of the boundary.

The method of deriving of these results is based on the explicit construction of the fundamental solution matrix for a generalized Cauchy-Riemann system with quaternion parameter (depending on a positive parameter) vanishing as the parameter tends to infinity on T when the pole of the fundamental solution lies in the half-space $y_3 > 0$. Following Lavrent'ev and Yarmukhamedov, we call the fundamental matrix of solutions with this property the **Carleman matrix** for the half-space [4],[5]. After the construction of the Carleman matrix in explicit form, the continuation and regularization formulas for the solution of the Cauchy problem can be written out as a generalized spatial Cauchy integral formula.

Statement of the problem. Given are the Cauchy data for the solution of system (1) on the surface S:

$$F(y)|_S = g(y), \;\; y \in S \tag{2}$$

where S is a part of the boundary of the domain Ω, $g(y) = \sum\limits_{k=1}^{3} g_k(y) i_k$ is a given continuous quaternion-valued function.

For $\alpha-$ hyperholomorphic function the quaternionic left Cauchy integral formula is defined (see [6, Subsection 4.15]):

Teorema. *Let Ω is a bounded simply connected domain in R^3 with boundary $\partial\Omega$ composed of a compact connected part T of the plane $y_3 = 0$ and a smooth Lyapunov surface S lying in the half-space $y_3 > 0$, with $\bar{\Omega} = \Omega \cup \partial\Omega$, $\partial\Omega = S \cup T$.. Let $f \in C^p(\Omega, H(C))$. Then*

$$K_\alpha^\sigma[f](x) := -\int\limits_S \widetilde{K}_\alpha^x [n_\tau f(\tau)] = f_\sigma(x), x \in \Omega. \tag{3}$$

where

(1) If $\alpha = \alpha_0 \in C$, then

$$\widetilde{K}^x_\alpha[f](\tau) := K_{\alpha_0}(x-\tau)f(\tau). \quad (4)$$

(2) If $\alpha \notin \overline{\Re}, \overrightarrow{\alpha}^2 \neq 0$, then

$$\widetilde{K}^x_\alpha[f](\tau) := \frac{1}{2\sqrt{\overrightarrow{\alpha}^2}}K_{\xi_+}(x)f(\tau)\sqrt{\overrightarrow{\alpha}^2} + \overrightarrow{\alpha} + \frac{1}{2\sqrt{\overrightarrow{\alpha}^2}}K_{\xi_-}(x)f(\tau)\sqrt{\overrightarrow{\alpha}^2} - \overrightarrow{\alpha}. \quad (5)$$

(3) If $\alpha \notin \overline{\Re}, \overrightarrow{\alpha}^2 = 0$, then

$$\widetilde{K}^x_\alpha[f](\tau) := K_{\alpha_0}(x)f(\tau) + \frac{\partial}{\partial\alpha_0}[K_{\alpha_0}](x)f(\tau)\overrightarrow{\alpha}. \quad (6)$$

(4) If $\alpha \in \Re, \alpha_0 \neq 0$, then

$$\widetilde{K}^x_\alpha[f](\tau) := \frac{1}{2\alpha_0}K_{2\alpha_0}(x)f(\tau)\alpha + \frac{1}{2\alpha_0}[K_{\alpha_0}](x)f(\tau)\overrightarrow{\alpha}. \quad (7)$$

(5) If $\alpha \in \Re, \alpha_0 = 0$, then

$$\widetilde{K}^x_\alpha[f](\tau) := K_0(x)f(\tau) + \Phi_0(x)f(\tau)\alpha. \quad (8)$$

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Essential spectrum of a 3×3 operator matrix with non compact perturbation

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Block operator matrices are matrices the entries of which are linear operators between Banach or Hilbert spaces [1]. They arise in various areas of mathematics and its applications: in systems theory as Hamiltonians, in the discretization of partial differential equations as large partitioned matrices due to sparsity patterns, in saddle point problems in non-linear analysis, in evolution problems as linearizations of second order Cauchy problems, and as linear operators describing coupled systems of partial differential equations. Such systems occur widely in mathematical physics, e.g. in fluid mechanics, magnetohydrodynamics, and quantum mechanics. In all these applications, the spectral properties of the corresponding block operator matrices are of vital importance as they govern for instance the time evolution and hence the stability of the underlying physical systems.

In the present note we investigate the essential spectrum of a 3×3 operator matrix with non-compact perturbation. This operator is associated with a lattice system describing two identical bosons and one particle, another nature in interactions, without conservation of the number of

Пространством основных функций$D = D(G) = C_0^\infty(G)$ называется векторное пространство функций неограниченное число раз дифференцируемых и обладающих компактными носителями. Обобщенная функция f определяется как непрерывный линейный функционал над D.

Будем говорить, что обобщенная функция f имеет порядок сингулярности не более чем k, если она непрерывно продолжается в пространство $C_0^k(G)$, k раз непрерывно дифференцируемых функций. Если, кроме того, f не продолжается в $C_0^{k-1}(G)$, то говорят, что порядок сингулярности обобщенной функции f равен точно k.

Теперь сформулируем основные результаты.

Теорема 1. *Пусть $f \in D'(G)-$ обобщенная функция порядка сингулярности $k : 0 \le k \le m$. Тогда для любого $\varepsilon > 0$ существует открытое множество U_ε с мерой Лебега $m(U_\varepsilon) < \varepsilon$ такое, что решение $u(x)$ уравнения $P(D)\,u = f$ принадлежит классу C^{m-k} на компактных подмножествах разности $G \backslash U_\varepsilon$.*

Теорема 2. *Пусть $f \in D'(G)-$ обобщенная функция порядка сингулярности $k : 0 \le n - m + k + \alpha < n$, $\alpha > 0$. Тогда для любого $\varepsilon > 0$ существует открытое множество U_ε с ёмкостью $cap_{n-m+k+\alpha}(U_\varepsilon) < \varepsilon$ такое, что решение $u(x)$ уравнения $P(D)\,u = f$ принадлежит классу C^α на компактных подмножествах разности $G \backslash U_\varepsilon$.*

В этих теоремах рассматривается только слабое решение, связанные с потенциалами Рисса. Оно определено однозначно и единственно.

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Нигде не дифференцируемые квазианалитические функции Гончара

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В классе непрерывных функций $C[a, b]$ С.Н. Бернштейн [1] определил класс квазианалитических функций в терминах быстрой полиномиальной аппроксимации.

Пусть f – функция, определенная и непрерывная на отрезке $\Delta = [a, b]$ действительной прямой $\mathbb{R}$, $e_m(f)$ – наименьшее отклонение функции f на отрезке Δ от полиномов степени не выше m:

$$e_m(f) = \inf_{\{p_m\}} \|f - p_m\|_\Delta \,,$$

где $\|\cdot\|_\Delta$ – максимальная норма и нижняя грань берется в классе всех полиномов степени не выше m.

Функция $f \in C(\Delta)$ голоморфно продолжается в некоторую комплексную окрестность $U \subset \mathbb{C}$ отрезка Δ тогда и только тогда, когда

$$\overline{\lim_{m\to\infty}} \sqrt[m]{e_m(f)} < 1. \qquad (1)$$

Если мы условие (1) заменим условием

$$\underline{\lim_{m\to\infty}} \sqrt[m]{e_m(f)} < 1, \qquad (2)$$