DISKRET HARDI OPERATORI NORMASI UCHUN BAHOLAR

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Anotatsiya. Ushbu maqolada Diskret Hardi tengsizligi, Diskret Hardi tengsizligi haqida dastlabki tushunchalar, Diskret Hardi tengsizligi tadbig‘iga oid dastlabki masalalar, bugunga qadar ilmiy-izlanishlar natijasi keltirilgan. Shu bilan bir qatorda Diskret Hardi tengsizligining parametri uchun muhim baholashlar keltirib chiqarilgan. Diskret Hardi operatori normasi baholab ko‘rsatilgan.

Kalit so‘zlar. Diskret Hardi tengsizligi, Diskret Hardi operatori, oddiy differensial tenglama, Hardi tengsizligi parametri, Diskret Hardi operatori normasi.

EVALUATIONS FOR THE DISCRETE HARDY OPERATOR NORM

Anotation. This article presents Discrete Hardy's inequality, initial concepts of Discrete Hardy's inequality, preliminary issues related to the application of Discrete Hardy's inequality, and the results of scientific research until today. In addition, important estimates for the parameter of Discrete Hardy's inequality have been derived. The norm of the discrete Hardy operator has been evaluated.

Keywords. Discrete Hardy inequality, Discrete Hardy operator, ordinary differential equation, parameter of Hardy inequality, norm of Discrete Hardy operator.

Ushbu maqolada Diskret Hardi tengsizligi, Diskret Hardi operatori, ularning qo‘llanilish sohalari o‘rganilgan. Diskret Hardi operatorini o‘rganishimiz uchun avvalo uni funksional va garmonik tahlil qilishimiz kerak bo‘ladi. Xususan so‘nggi yillarda Diskret Hardi tengsizligini o‘rganish bo‘yicha ko‘pgina yutuqlarga erishildi. Diskret Hardi operatori mavzusi shular jumlasidandir. Ushbu borada bugungi kungacha o‘rganilgan bir nechta masalalarni ko‘rib chiqamiz va tahlil qilamiz. Hardi tengsizligidan foydalangan holda turli sohalar, xususan chiziqli integral tenglamalar, operatorlar nazariyasi, operatorlarni chiziqli va chegaralanganlikka tekshirishda [1-6] muhim natijalarga erishilgan. Dastlabki qo‘yilgan masalalardan biri, bu bir o‘lchovli Hardi tipidagi tengsizlikning haqiqiyligi va uning oddiy differensial tenglamaning yechilishi orasidagi bog‘liqliqlikni ifodalovchi Chen masalasi hisoblanadi. Chenning natijalari Bisak, Kafner va Tribel Mukenhaptlarning ko‘pgina ishalariga asoslangan edi. Chen o‘zining ilmiy izlanishlari natijasida olgan natijasini ifodalash uchun zarur bo‘lgan tushunchalarni kiritamiz: Quyidagi teorema orqali Hardi tengsizligi parametrining yuqoridan chegaralanish baholarni olamiz.

Teorema1. Hardi tengsizlikdagi A parametr quyidagicha baholashni qanoatlantiradi. Yuqoridan baholanish:

$$A \le \inf_{x\in A[1,N]} \left(\sup_{n\in[1,N]} II_n^*(x) \right)^{\frac{1}{p^*}} = \inf_{x\in A[1,N]} \left(\sup_{n\in[1,N]} I_n^*(x) \right)^{\frac{1}{p^*}}. \qquad (1)$$

Quyidan baholash:

$$A \ge \sup_{x\in A[1,N]} \|x\|_{l^p(v)}^{\frac{p}{q}-1} \left(\inf_{n\in[1,N]} II_n(x) \right)^{\frac{p-1}{q}}$$

$$= \sup_{x\in A[1,N]} \|x\|_{l^p(v)}^{\frac{p}{q}-1} \left(\inf_{n\in[1,N]} I_n(x) \right)^{\frac{p-1}{q}} \qquad (2)$$

Asosiy quyi baholar esa yanada sodda yo‘l bilan olinadi. Har qanday $n \in [1,\infty)$ uchunbiz quyidagi ketma ketlikni tanlashimiz mumkin:

$$x_i^{(n)} = \begin{cases} \hat{v}_i, & 1 \le i \le n \\ 0, & n < i < \infty \end{cases}$$

Ko‘rinib turibdiki, bu yerda $x^{(n)} \in A_0[1,\infty)$ bo‘ladi. Shundan

$$A = \sup_{x\in A[1,N]} \frac{\left[\sum_{n=1}^N u_n \left(\sum_{i=1}^n x_i\right)^q\right]^{\frac{1}{q}}}{\left(\sum_{n=1}^N u_n x_n^p\right)^{\frac{1}{p}}} = \sup_{x\in[1,N]} \frac{\|Hx\|_{l^q(u)}}{\|x\|_{l^p(v)}}$$

munosabatga muvofiq quyidagi o‘rinlidir

$$A \ge \sup_{n\in[1,\infty)} \frac{\left[\sum_{i=1}^\infty u_i \left(Hn^{(n)}(i)\right)^q\right]^{\frac{1}{q}}}{\left[\sum_{i=1}^\infty v_i \left(x_i^{(n)}\right)^p\right]^{\frac{1}{p}}} =$$

$$= \sup_{n\in\{1,\infty)} \left(\sum_{i=1}^n \hat{v}_i\right)^{\frac{1}{p^*}} \left(\left(\sum_{i=1}^n \hat{v}_i\right)^{-q} \left(\sum_{i=1}^{n-1} u_i \left(H\hat{v}(i)\right)^q\right) + \sum_{i=n}^\infty u_i \right)^{\frac{1}{q}} \ge B$$

Lemma 1. a va b manfiy bo‘lmagan biror ketma-ketlik uchun quyidagi shart bajarilsa,

$$\sum_{k=i}^\infty a_k \le \sum_{k=i}^\infty b_k \qquad (\forall i = 1,2,\dots)$$

u holda shunday biror-bir manfiy bo‘lmagan $\{c_k\}$ ketma-ketlik mavjudki quyidagi bajariladi:

$$\sum_{k=1}^{\infty} a_k c_k \le \sum_{k=1}^{\infty} b_k c_k \qquad (3)$$

Isbot. Ma'lumki $c_0 = 0$. Qismlar bo'yicha yig'indilardan foydalanib, (3) tengsizlikning chap tomonini quyidagicha yozib olamiz:

$$\sum_{k=1}^{\infty} a_k c_k = \sum_{k=1}^{\infty} \left(\sum_{i=1}^{k} c_i - c_{i-1} \right) a_k$$

$$= \sum_{i=1}^{\infty} \left(\sum_{k=i}^{\infty} a_k \right) (c_i - c_{i-1}) \le \sum_{i=1}^{\infty} \left(\sum_{k=i}^{\infty} b_k \right) (c_i - c_{i-1}) = \sum_{k=1}^{\infty} b_k c_k$$

Lemma isbotlandi.

Lemma 2. Har qanday manfiy bo'lmagan haqiqiy $f(x)$ funksiya uchun quyidagi tengsizlik o'rinlidir:

$$\left(\int_0^{\infty} \frac{1}{x^{q-r}} \left(\int_0^x f(t)dt \right)^q \right)^{\frac{1}{q}} \le k_{p,q} \left(\frac{p^*}{q} \right)^{\frac{1}{q}} \left(\int_0^{\infty} f^p(t)dt \right)^{\frac{1}{p}} \qquad (4)$$

Bunda $r = \frac{q}{p} - 1$ va $k_{p,q}$ optimal o'zgarmasdir. Optimal o'zgarmas mavjud bo'lishi uchun $f(x)$ funksiya quyidagicha aniqlanishi kerak.

$$f(x) = \frac{c}{(d \cdot x^r + 1)^{\frac{r+1}{r}}} \qquad (5)$$

(4) ifodada c va d lar nomanfiy o'zgarmaslardir.

Asosan yuqori baholarning omilini ko'rsatish uchun biz quyi baholarni ham shu yo'l bilan olishga harakat qilamiz [6-12]. Lemma 2 dan quyidagi ekstremal funksiyalarni qaraymiz:

$$f(x) = \frac{c}{(dx^r + 1)^{\frac{r+1}{r}}} \qquad va \qquad \int_0^x f(t)dt = \frac{cx}{(dx^r + 1)^{\frac{1}{r}}}$$

Bu yerda c va d lar ixtiyoriy musbat o'zgarmaslardir. Ushbu shartlarni kiritamiz:

$$u_n = n^{-\frac{q}{p^*}} - (n+1)^{-\frac{q}{p^*}}, \qquad v_n \equiv 1$$

va

$$x_n = \frac{cn}{(n^r + d)^{\frac{1}{r}}} - \frac{c(n-1)}{((n-1)^r + d)^{\frac{1}{r}}}.$$

Shubhasizki, x o'zgaruvchining qandayligi

$$\int_0^x f(t)dt$$

integralning ko'rinishidan kelib chiqadi. Bu holda biz $B=1$ deb olamiz. Bu yerda biz c va d larni ixtiyoriy tanalashimiz mumkin. Lekin c va d qanday tanlanmasin integralni baholashdagi aniqlik yo'qoladi [10-18]. Ammo to'g'ridan- to'g'ri qilingan hisob kitoblar orqali biz bu yo'qotish $\frac{c}{d}$ ratsional sonning qiymati nolga intilganida mutlaqo ahamiyatsizligini keltrib chiqara olamiz. Umumiylikni saqlagan holda biz $c=1$ ni tanlab olamiz va d musbat va yetarlicha katta son bo'lsin . Keyin ushbu

$$\left[\sum_{n=1}^{N} u_n\left(\sum_{i=1}^{n} x_i\right)^q\right]^{\frac{1}{q}} \leq A\left(\sum_{n=1}^{N} v^n x_n^p\right)^{\frac{1}{p}} \tag{6}$$

esa (6) tengsizlikning chap va o'ng tomonini qayta hisoblaymiz.

$$\sum_{n=1}^{\infty} x_n^p = \sum_{n=1}^{\infty}\left[\frac{n}{(n^r+d)^{\frac{1}{r}}} - \frac{(n-1)}{((n-1)^r+d)^{\frac{1}{r}}}\right]^p = \sum_{n=1}^{\infty}\left[\int_{n-1}^{n} \frac{d}{(x^r+d)^{\frac{1}{r+1}}} dx\right]^p \leq$$

$$\sum_{n=1}^{\infty}\int_{n-1}^{n}\left(\frac{d}{(x^r+d)^{\frac{1}{r+1}}}\right)^p dx = r^{-1} d^{\frac{1-p}{r}} B\left(\frac{1}{r}, \frac{q-1}{r}\right). \tag{7}$$

(6) tengsizlikning chap tomonini hisoblash biroz murakkab bo'lib, dastlab biz N yetarlicha katta butun son mavjud deb olamiz. Bunda

$$\int_{N}^{\infty} (x)^{-\frac{q}{p^*-1}}\left[\frac{x^q}{(x^r+d)^{\frac{q}{r}}}\right] dx \leq \int_{1}^{\infty} (x+1)^{-\frac{q}{p^*-1}}\left[\frac{x^q}{(x^r+d)^{\frac{q}{r}}}\right] dx \tag{8}$$

Munosabat o'rinli bo`ladi. Aks holda quyidagi tengsizlikdan

$$\int_{N}^{\infty} (x)^{-\frac{q}{p^*-1}}\left[\frac{x^q}{(x^r+d)^{\frac{q}{r}}}\right] dx \leq \int_{N}^{\infty} (x)^{-\frac{q}{p^*-1}} dx = \frac{p^*}{q} N^{-\frac{q}{p^*}}.$$

N ning mavjudligi va ixtiyoriyligi kelib chiqadi. Chunki (8) ning chap tomoni nolga o'ng tomoni esa $N \uparrow \infty$. bu esa N ni yetarlicha katta olishimizga imkon beradi. Keyin esa (8) munosabatning chap tomonini hisoblashimiz mumkin bo'ladi. $s^{-1} = d^{-1}x^r + 1$ dan foydalanib biz quyidagiga ega bo'lamiz:

$$\int_{N}^{\infty} (x)^{-\frac{q}{p^*-1}}\left[\frac{x^q}{(x^r+d)^{\frac{q}{r}}}\right] dx = r^{-1} d^{\frac{-p}{rq^*}} B\left(\frac{1+r}{r}, \frac{q-r-1}{r}, \frac{d}{N^r+d}\right), \tag{9}$$

Bu yerda $B(a,b,x)$ to'liq bo'lmagan Betta funksiyasi:

$$B(a,b,x)=\int_0^x s^{a-1}(1-s)^{b-1}ds\ .$$

(8) va (9) uchun biz o‘rtacha qiymat teoremasini qo‘llagan holda biz quyidagiga ega bo‘lishimiz mumkin:

$$\int_1^\infty \left[(x)^{-\frac{q}{p^*}}-(x+1)^{-\frac{q}{p^*}}\right]\frac{x^q}{(x^r+d)^{\frac{q}{r}}}dx \geq$$

$$\geq \int_1^\infty \frac{q}{p^*}(x+1)^{-\frac{q}{p^*-1}}\left[\frac{x^q}{(x^r+d)^{\frac{q}{r}}}\right]dx \geq$$

$$\geq d^{\frac{-p}{rq^*}}\frac{q}{p^*}r^{-1}B\left(\frac{1+r}{r},\frac{q-r-1}{r},\frac{d}{N^r+d}\right) \qquad (10)$$

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